Discrete-event neural modeling: Toward linking biological and cognitive levels

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Questions

Discrete models of leaky integrate-and-fire neurons

- Discrete-time/event representation of leaky integrate-and-fire neurons for digital computers?
- A new coding scheme of input trains?
- Computation and information simplification?

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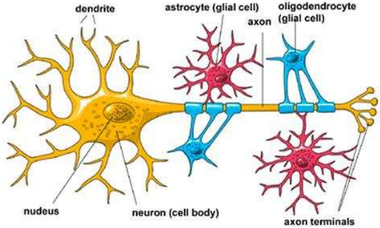
Neural entities

- Brain = nerve cells (*neurons*) and glial cells (support and protect neurons)



Neurotransmission

- Electrical events
  - Flow of electrical charges (+/- particles: ions) thru voltage-gated ion channels
  - Info = Variations in voltage, current, frequency, phase, or duration

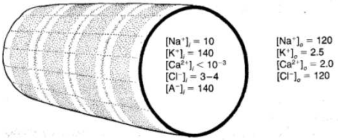
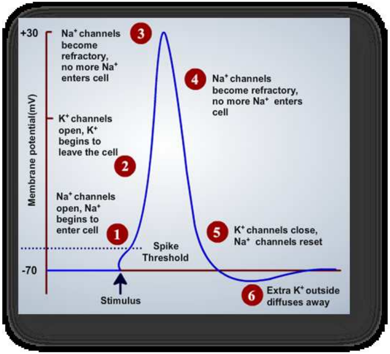
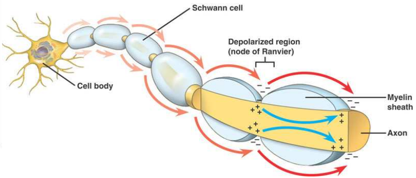


Figure 4-13 Concentrations of common ions are very different inside and outside a vertebrate skeletal muscle cell. The concentrations shown are in millimoles per liter. The concentration given for intracellular Ca^{2+} is for the free, unbound, and unsequestered ion in the myoplasm. Because the list of ions is incomplete, the totals do not balance out perfectly. $[A^-]_i$ represents the molar equivalent negative charges carried by various impermeant anions.

Randall et al. (1997) Eckert Animal Physiology, WH Freeman and Company.

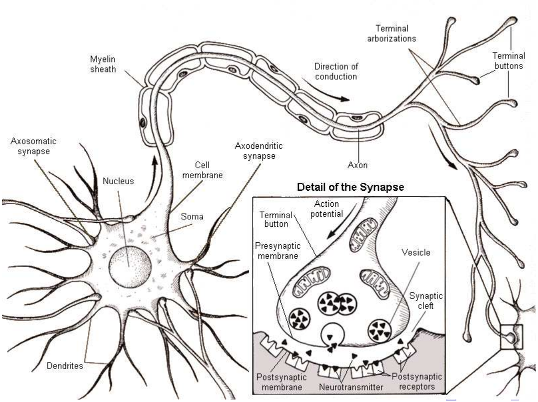
Neurotransmission





Neurotransmission

- Chemical messengers
  - Neurotransmitters released at synapses by electric. events
  - Info = Variations in concentration, types, compatibility with receptors (1/0), excitatory/inhibitory



Characteristics

- Discrete dynamic system
- Leaks
  - Diffusion of ions through the membrane
  - Time-dependent memory
  - Non trivial input patterns
- Weights (myelination of axon, #neurotransmitters/synapses...)

Discrete-time neuron model

In a usual *leaky integrate-and-fire discrete model*, at time $t \in \mathbb{N}$, the *membrane potential* $s(t) \in \mathbb{R}$ of a neuron consists of:

$$s(t) = \begin{cases} rs(t-1) + \sum_{j=1}^{m} w_j x_j(t) & \textit{if } s(t-1) < \tau \\ 0 & \textit{otherwise} \end{cases}$$

Spike emission $x(t)$ depends on *threshold* $\tau \in \mathbb{R}^+$:

$$x(t) = \begin{cases} 1 & \textit{if } s(t-1) \geq \tau \\ 0 & \textit{otherwise} \end{cases}$$

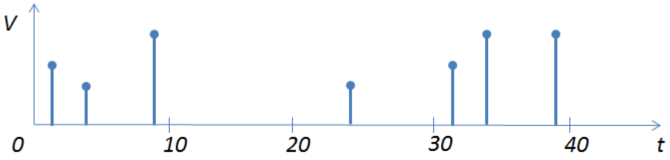
Discrete-time neuron model

$$\begin{pmatrix} s_1(t) \\ s_2(t) \\ \vdots \\ s_m(t) \end{pmatrix} = (w_1 \dots w_m) \begin{pmatrix} x_1(t-\sigma) & \cdots & x_1(t-1) & x_1(t) \\ x_2(t-\sigma) & \cdots & x_2(t-1) & x_2(t) \\ \vdots & \ddots & \vdots & \vdots \\ x_m(t-\sigma) & \cdots & x_m(t-1) & x_m(t) \end{pmatrix} \begin{pmatrix} r^\sigma \\ \vdots \\ r \\ 1 \end{pmatrix}$$

- error ϵ
- elapsed time $e \leq \frac{ln(\epsilon/\Sigma_j w_j)}{ln(r)}$
- *integration time window* $\sigma = \frac{ln(\epsilon/\Sigma_j w_j)}{ln(r)}$, for $\epsilon = 1\%$, $\Sigma_j w_j = 1$, and $r = 50\%$: $\sigma = 6.64$ or, as $t \in \mathbb{N}$, $\sigma = \lceil \sigma \rceil = 7$

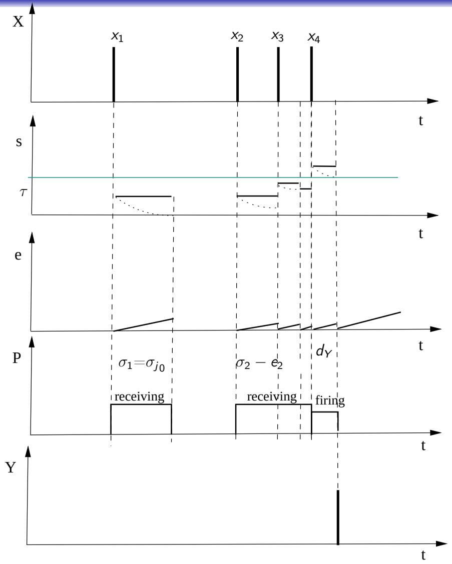
Discrete-events and activity

Event set $\xi = \{ev = (t, v) \mid t \in [0, T] \wedge v \in V\}$

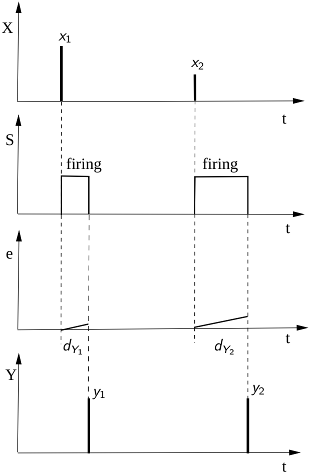


Instantaneous activity
$a_\xi(t) = |\{ev \in \xi \mid t \in [t', t' + T] \wedge v \in V\}|$
Accumulated activity $a_\xi(T) = \Sigma_{t \in [t', t'+T]} a_\xi(t)$

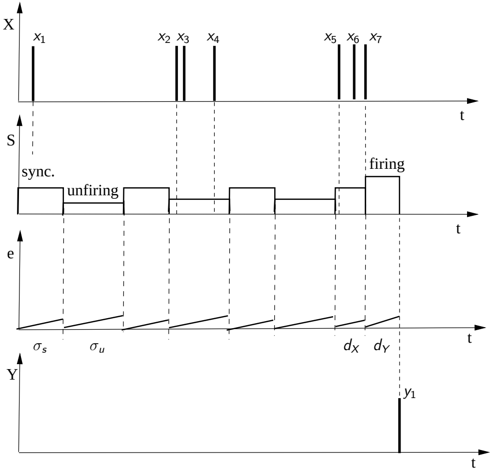
Discrete-event neuron model



Delay neuron



Synchronization neuron model



Bursts

- "Burst firing (...) [consists] of trains of two or more spikes occurring within a relatively short interval and followed by a prolonged period of inactivity."

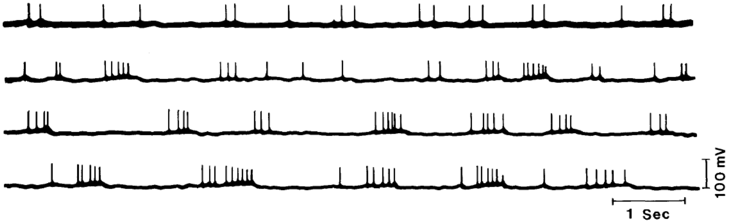
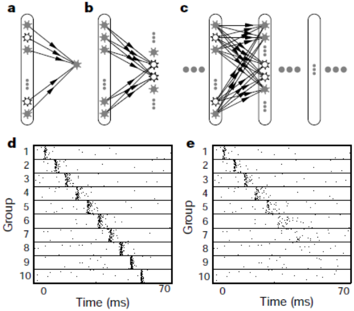


Figure 11. Effect of intracellular calcium injection on the firing pattern of nigral DA cells. In the first few minutes following impalement with a calcium-containing electrode, the stabilized DA cell demonstrates its typical slow, single spike firing pattern (*top trace*). As calcium leaks from the electrode into the cell, the pattern slowly changes over the next 10 to 20 min into a burst-firing pattern (*second* through *fourth trace*).

Grace and Bunney (The journal of neuroscience, 1984)

Assembly synchronization



Diesmann (Nature 1999): synchronization accuracy in cell assemblies

(Intermediate) Iterative system specification

$$S_f = (X, \Omega_G, Y, T, Q, \delta, \lambda)$$

X, Y the input/output sets
T the time base
Q the set of total states
Ω_G the *set of input segment (or admissible set of) generators such that* $\Omega_G \subseteq X^T$ with X^T the set of *all input segments* (all functions from *time set* T to *input set* X)
$\delta : Q \times \Omega_G \rightarrow Q$ the *single segment transition function*
$\lambda : Q \times X \rightarrow Y$ the *output function.*

(Intermediate) Iterative system specification

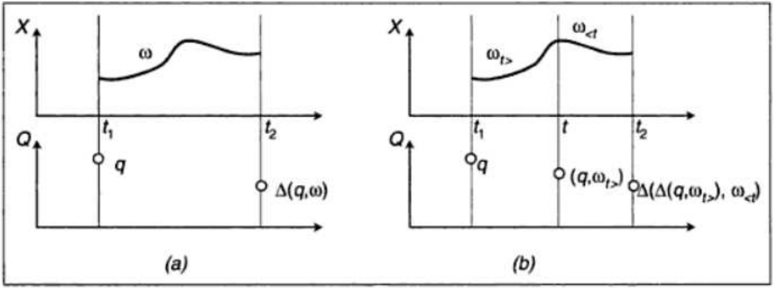


Figure 10 The composition property. (a) Results from a single composite experiment (b) Results from a two-part experiment.

BP Zeigler et al. (2000), Theory of modeling and simulation, Academic Press.

Maximal length segmentation (mls)

- Unic segmentation of trajectories
- Independent input-state-output segments

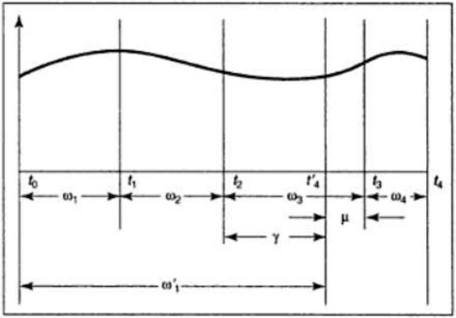
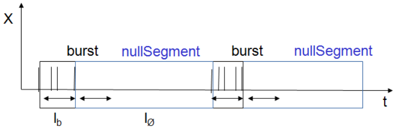


Figure 14 Initial segment of mls decomposition.

BP Zeigler et al. (2000), Theory of modeling and simulation, Academic Press.

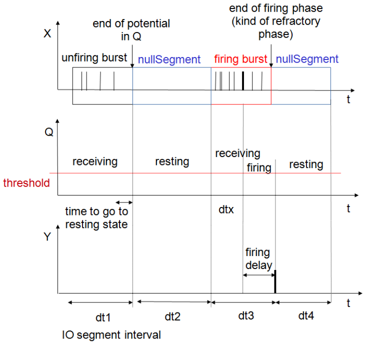
Bursty input segmentation

- Set of input segment generators: $\Omega_G = \{b, \emptyset\}$
- Set of input patterns (bursty segments): $\Omega_X = \{b\emptyset, \emptyset, b, \emptyset b, \ldots\}$
- Stop criterion $l_b < l_\emptyset$



Bursty input-state-ouput segmentation

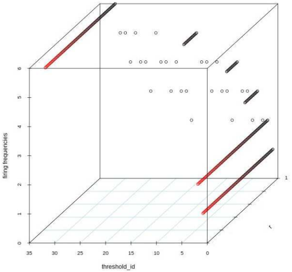
- Set of input segment generators: $\Omega_G = \{b, \overline{b}, \emptyset\}$
- Set of input patterns (bursty segments): $\Omega_X = \{b\overline{b}\emptyset bb, \overline{b}\emptyset b, ...\}$



Number of inputs to fire over an interval

- The neuron fires *if* a number of n_f inputs are received over a synchronization window of length σ_f (= accumulated activity $a_f(\sigma_f) = n_f$).
- σ_f is determined by

  $$1 + (n_f - 1)r^{\sigma_f} > \tau \Rightarrow \sigma_f = \frac{ln(\frac{\tau-1}{n_f-1})}{ln(r)} \text{ and } \tau < n_f$$
- also do not fire for less input at current time: $n_f - 1 < \tau$
- We find $\tau < n_f < \tau + 1$



Advantages of bursty neurons

- Discrete-events:
  - No error of state change computations
  - Simplification of computations/information
  - Explicit delays and time intervals
- Segmentations:
  - MIs (without state knowledge but does not satisfy IO segmentation match)
  - Iput-State-Output (more efficient and satisfies IO segmentation match)
- Perspectives:
  - Synchronization of assemblies
  - Delays (mental chronometry...)
  - Define morphism between discrete-event systems